Detectors

RIT Course Number 1051-465
Lecture Noise

Aims for this lecture

- learn to calculate signal-to-noise ratio
- describe processes that add noise to a detector signal
- give examples of how to combat noise

Lecture Outline

- signal-to-noise ratio
- noise sources
  - shot noise
    - signal
    - dark current
    - background
  - thermal noise (Johnson)
  - kTC
  - 1/f
  - read noise in the electronics (summation of many contributors)
  - electronic crosstalk
  - popcorn noise
  - pickup/interference
- how to combat noise, improving SNR

Introduction

- The ability to detect an object in an image is related to the amplitude of the signal and noise (and by contrast in some cases).
- The signal can be increased by collecting more photons, e.g.
  - building a bigger lens,
  - using more efficient optics,
  - having a more efficient detector.
- The noise can be reduced by making better electronics.

Signal-to-Noise Ratio

What is Signal? What is Noise?

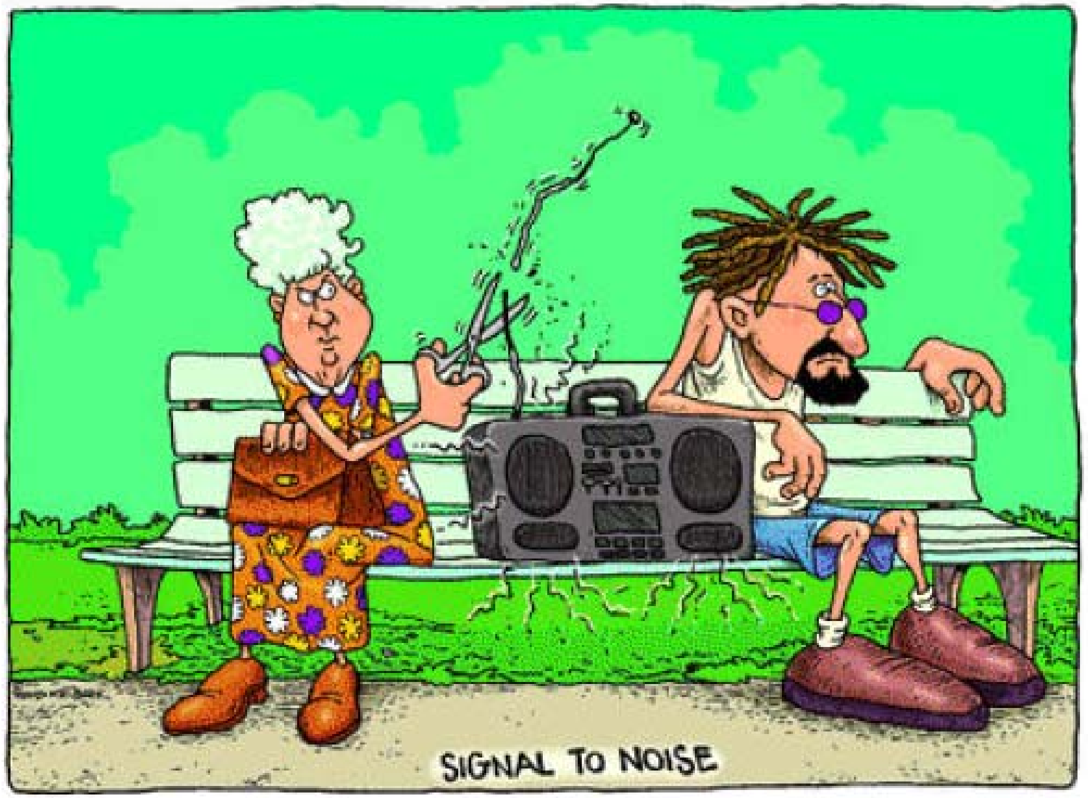


FIG. 1: Whether noise is a nuisance or a signal may depend on whom you ask. (Cartoon by Rand Kruback. Used with permission of Agilent Technologies.)

SNR and Sensitivity Definitions

- SNR is the ratio of detected signal to uncertainty of the signal measurement. Higher is better.
- Sensitivity is the flux level that corresponds to a given SNR for a particular system and integration time. Lower is better.

SNR and Sensitivity

- Both SNR and sensitivity depend on
  - signal
    - brightness of source
    - absorption of intervening material
      - gas, dust
      - particles in the atmosphere
      - optics
    - size of telescope
    - efficiency of detector
    - integration time
  - noise
    - detector read noise
    - detector dark current
    - background (zodiacal light, sky, telescope, instrument)
    - shot noise from source
    - imperfect calibrations

Signal: definition

- Signal is that part of the measurement which is contributed by the source.

$$S = \eta_{total} A \frac{\Delta \nu}{h \nu} F_{\nu} t QE \{e^{-}\}, \ where$$

$\mathrm{A} =$ area of aperture,

$\mathrm{QE} =$ quantum efficiency of detector,

$\Delta \nu =$ frequency bandwidth,

$\mathrm{F}_{\nu} =$ source flux,

$\eta_{\mathrm{total}} =$ total transmission, and

$\mathrm{t} =$ integration time.

Noise - definition

- Noise is that part of the measurement which is due to sources other than the object of interest.
- In sensitivity calculations, the “noise” is usually equal to the standard deviation.
- Random noise adds in quadrature.

$$N_{total} = \sqrt{\sum_i N_i^2}.$$

Sensitivity: definition

- Sensitivity is the flux level that corresponds to a given SNR.

$$Sensitivity_{desired\,SNR} = F_{\nu,desired\,SNR} = \frac{Noise}{\eta_{total} A \frac{\Delta\nu}{h\nu} tQE} SNR_{desired} \left\{ \frac{ergs}{cm^2\, s\, Hz} \right\}, where$$

where, A = area of aperture,

QE = quantum efficiency of detector,

$\Delta\nu$ = frequency bandwidth,

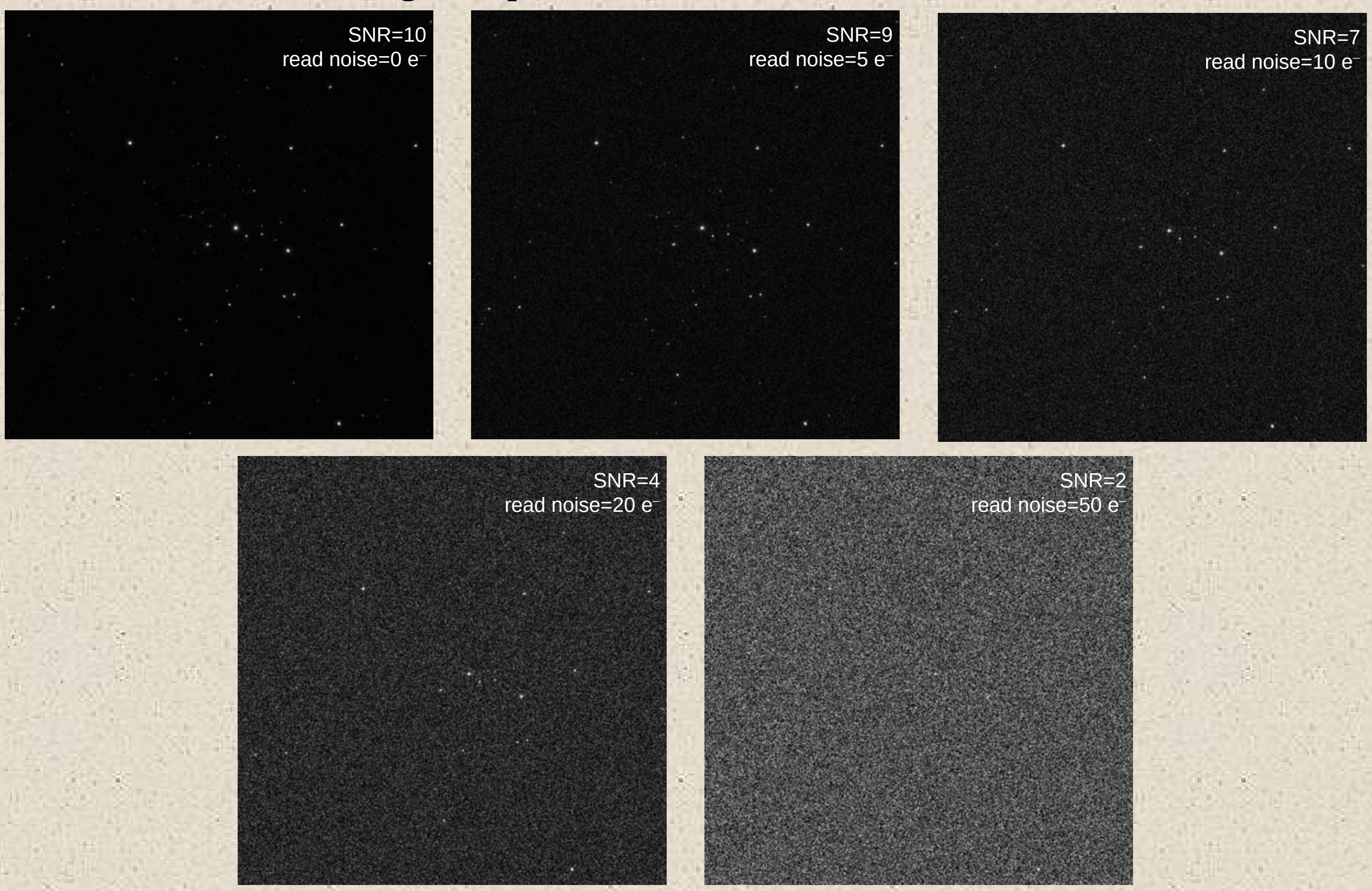
F_ν = source flux,

η_{total} = total transmission, and

t = integration time.

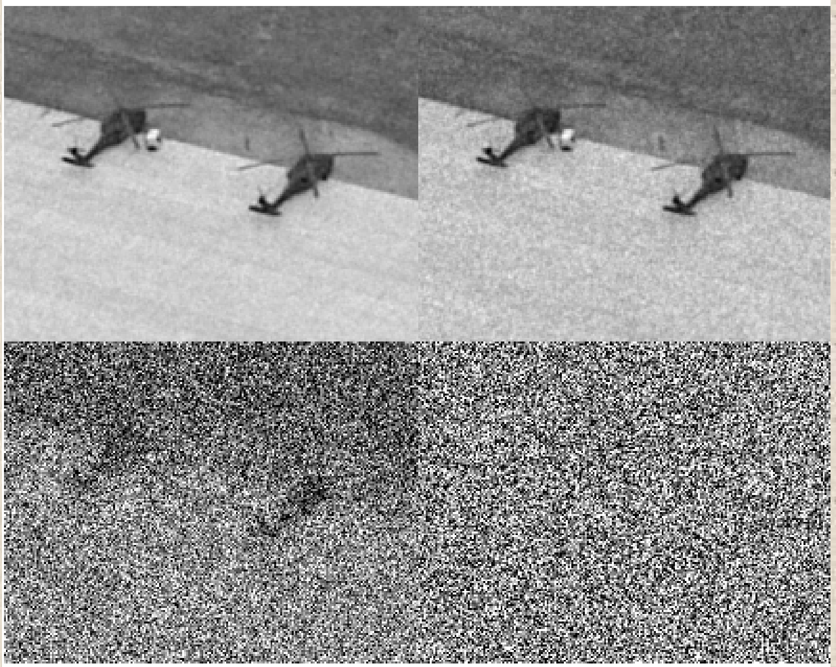
SNR Example

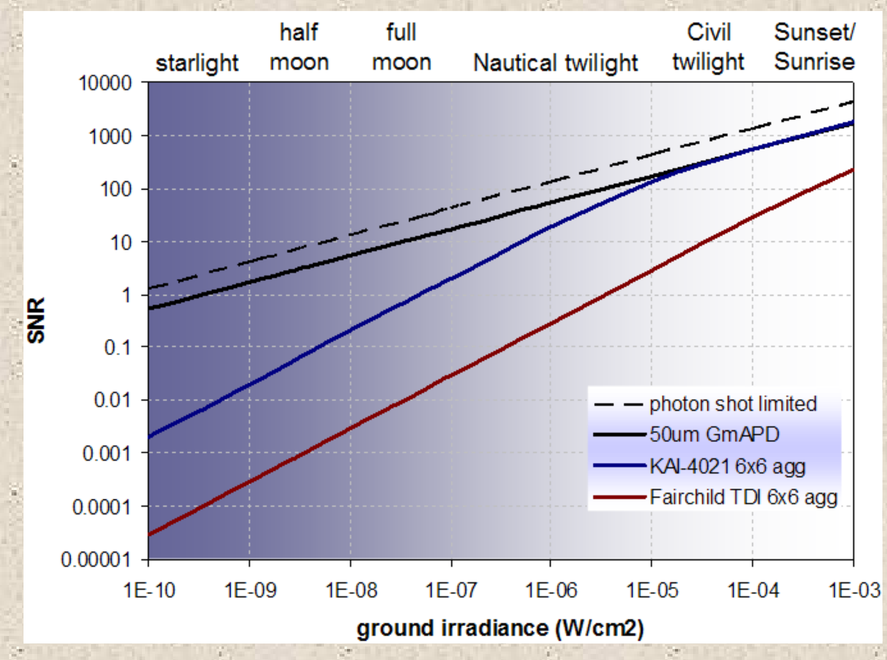
- These images show synthetic noise added to a real image of a star cluster obtained using Keck/LGSAO.
- The maximum signal has been normalized to 100 and shot noise has been added. SNR values are for brightest pixel.



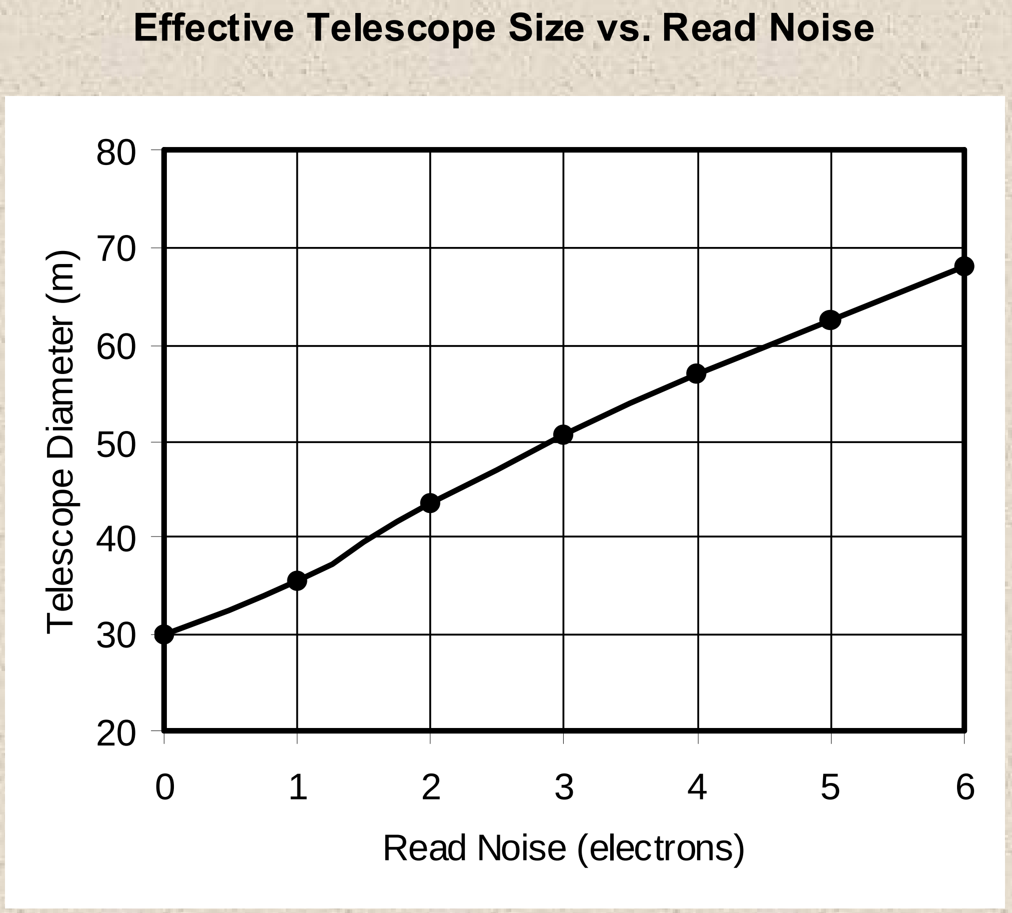
Imaging at Night

- Signal is diminished as light level is reduced.
- Read noise is very important for object identification in this case.





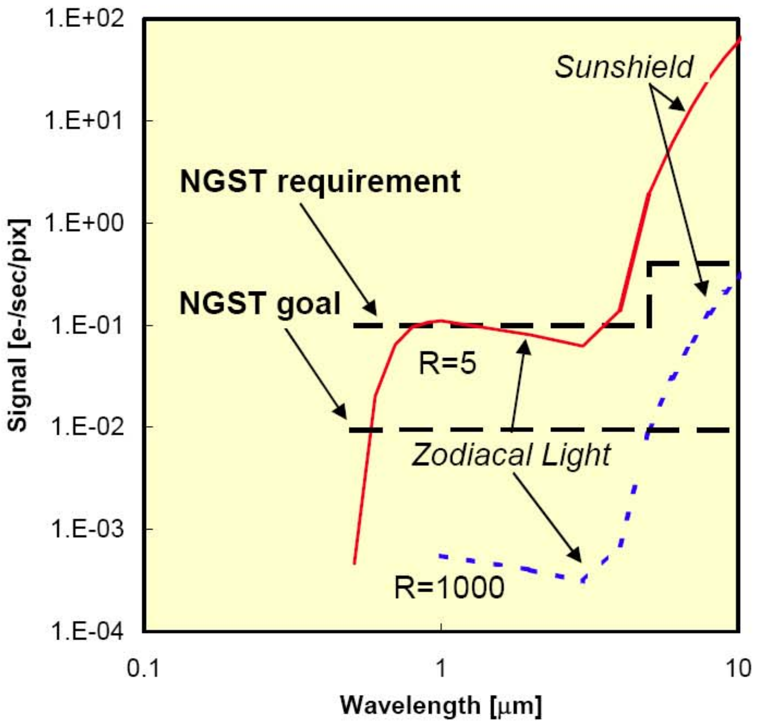
Read Noise vs. Telescope Size

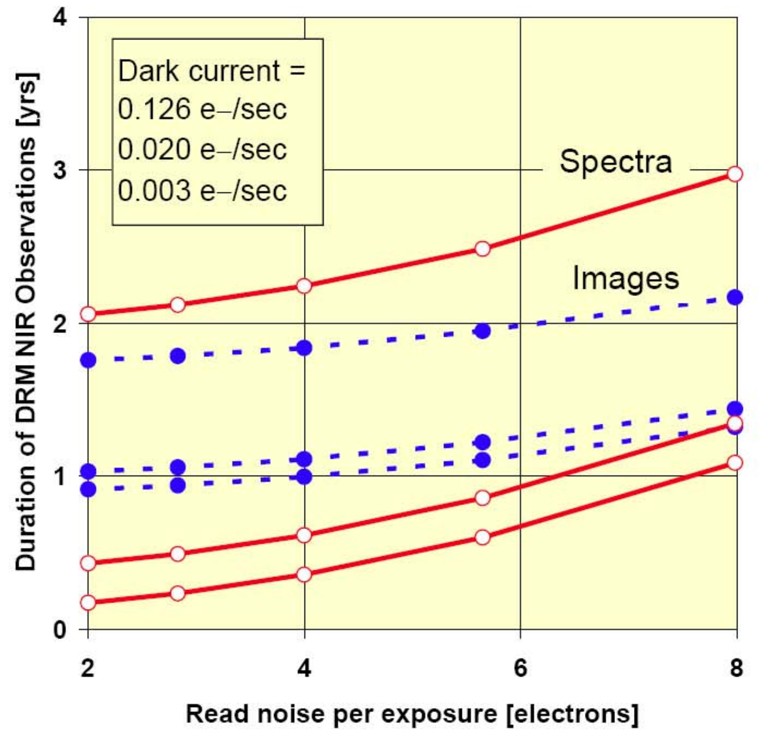


This plot shows a curve of constant sensitivity for a range of telescope diameters and detector read noise values in low-light applications. A 30 meter telescope and zero read noise detector would deliver the same signal-to-noise ratio as a 60 meter telescope with current detectors.

Noise and Performance

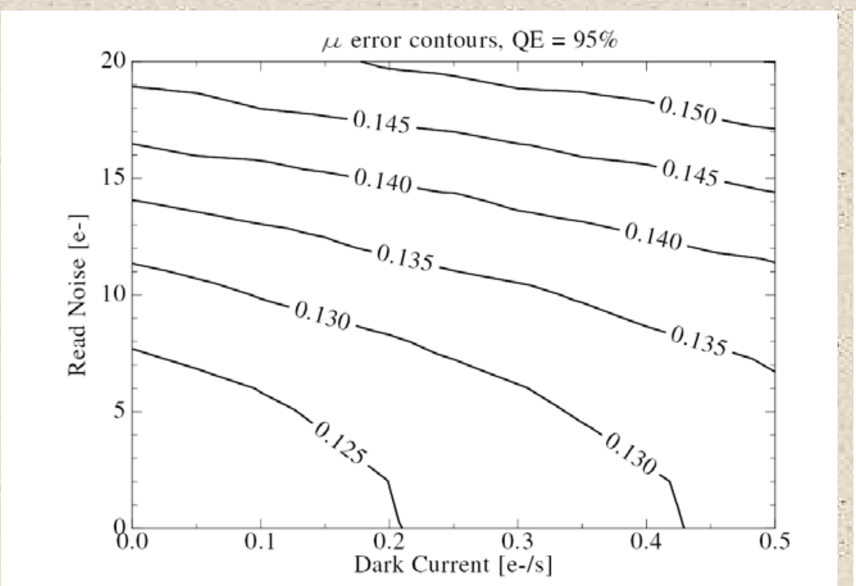
- These plots show system performance for NGST (now called JWST).
- Note that lower noise gives better performance.

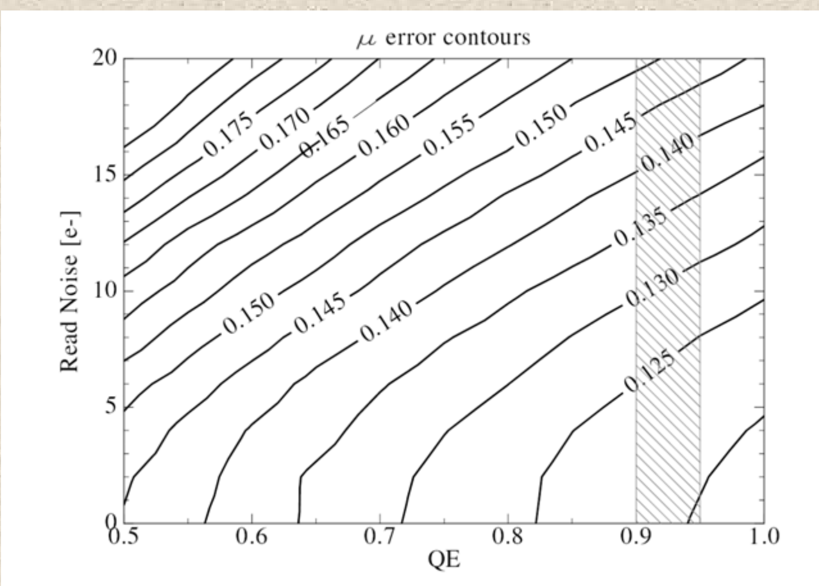




Tradeoff Example

- An improvement in SNR due to one detector property can allow one to relax the performance in another property.
- One last step in making such a trade is to convert improvements into dollars.





Joint Dark Energy Mission, Brown 2007, PhD Thesis

Noise Sources: Shot Noise in Signal

Shot Noise Described

- Photons arrive discretely, independently and randomly and are described by Poisson statistics.
- Poisson statistics tells us that the Root Mean square uncertainty (RMS noise) in the number of photons per second detected by a pixel is equal to the square root of the mean photon flux (the average number of photons detected per second).
- For example, a star is imaged onto a pixel and it produces on average 10 photo-electrons per second and we observe the star for 1 second, then the uncertainty of our measurement of its brightness will be the square root of 10 i.e. 3.2 electrons. This value is the ‘Photon Noise’. Increasing exposure time to 10 seconds will increase the photon noise to 10 electrons (the square root of 100) but at the same time will increase the ‘Signal to Noise ratio’ (SNR).
- In the absence of other noise sources the SNR will increase as the square root of the exposure time.

Photon Shot Noise

- The uncertainty in the source charge count is simply the square root of the collected charge.

$$N_{source} = \sqrt{S} = \sqrt{\eta_{total} A \frac{\Delta \nu}{h \nu} F_{\nu} t QE} \{e^{-}\}.$$

- Note that if this were the only noise source, then S/N would scale as $t^{1/2}$. (Also true whenever noise dominated by a steady photon source.)

Noise Sources: Shot Noise from Background

Noise - sources: Noise from Background

- Background photons come from everything but signal from the object of interest!
- Note that the noise contribution is simply the uncertainty in the background level due to shot noise.

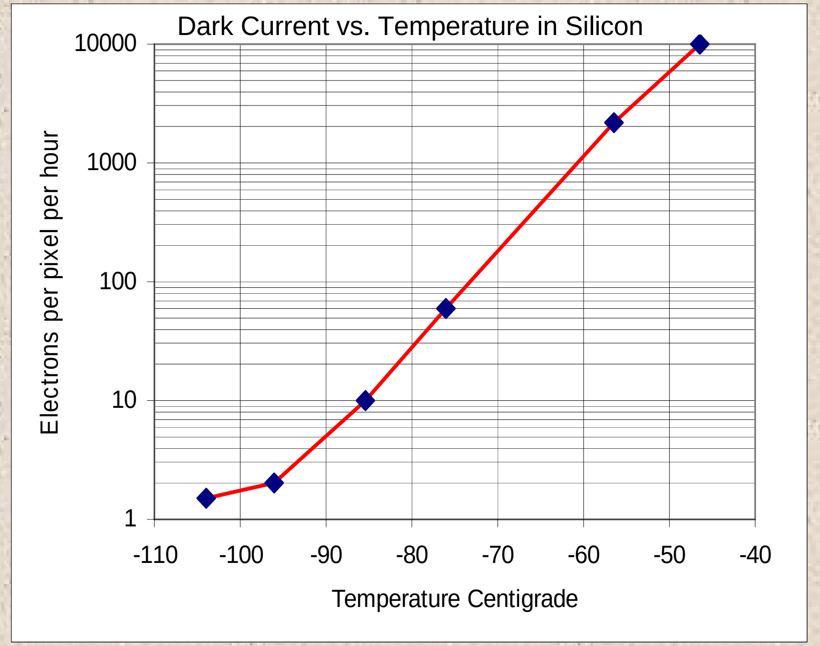
$$N_{back} = \sqrt{\dot{C}_{back} t} = \sqrt{\eta_{total} A \frac{\Delta \nu}{h \nu} F_{back,\nu} t QE} \{e^{-}\}.$$

Noise Sources: Shot Noise from Dark Current

Shot Noise of Dark Current

- Charge can also be generated in a pixel either by thermal motion or by the absorption of photons.
- The two cases are indistinguishable.
- Dark current can be reduced by cooling.

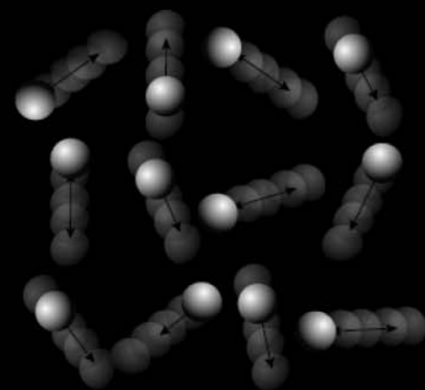
$$N_{dark} = \sqrt{\dot{C}_{dark} t} \; \{e^{-}\}.$$

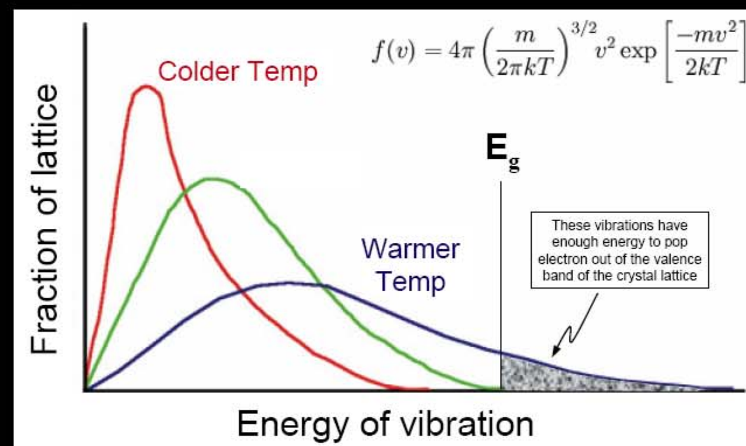


Dark Current Mechanism

Dark Current

Undesirable byproduct of light detecting materials

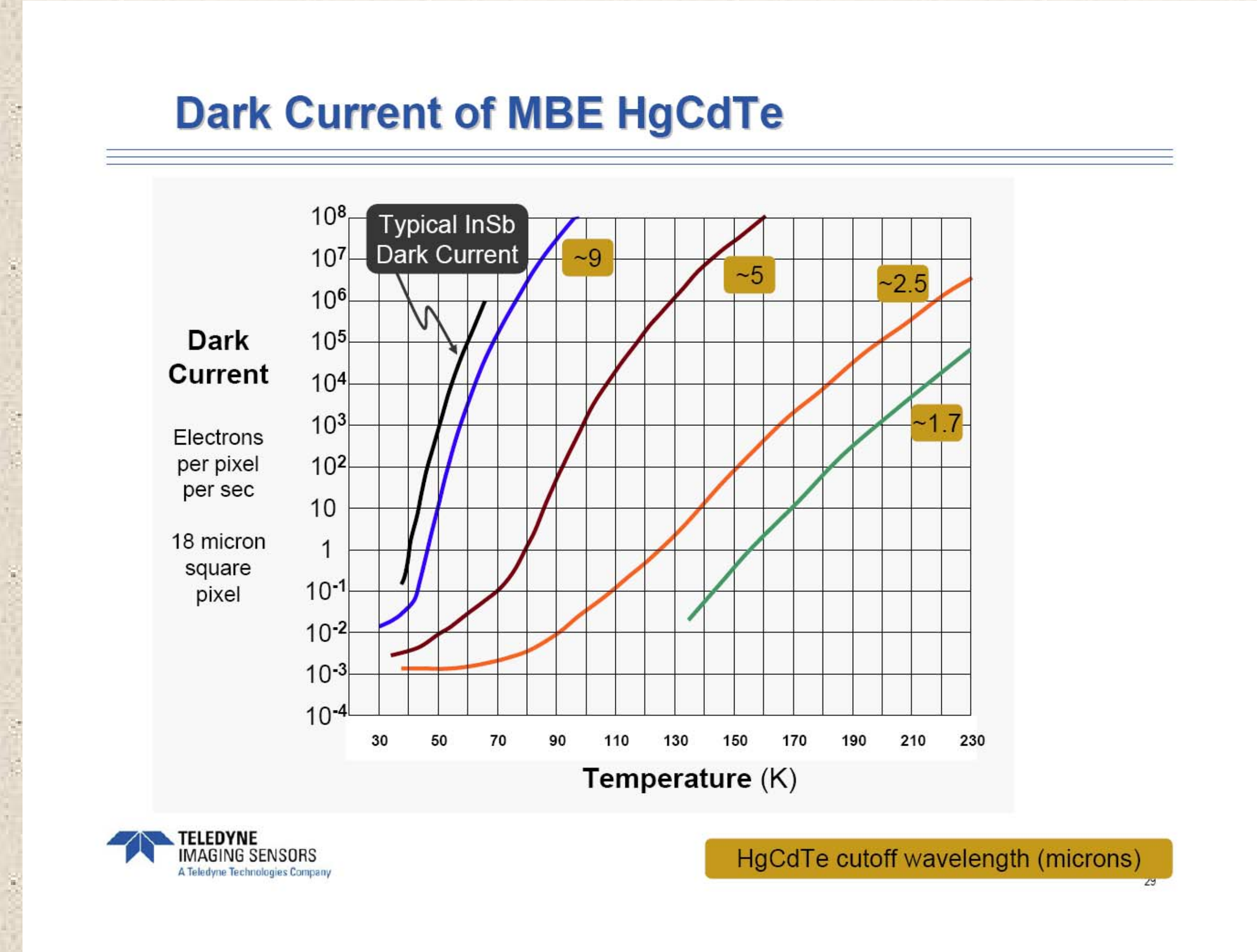




- The vibration of particles (includes crystal lattice phonons, electrons and holes) has energies described by the Maxwell-Boltzmann distribution. Above absolute zero, some vibration energies may be larger than the bandgap energy, and will cause electron transitions from valence to conduction band.
- Need to cool detectors to limit the flow of electrons due to temperature, i.e. the **dark current** that exists in the absence of light.
- The smaller the bandgap, the colder the required temperature to limit dark current below other noise sources (e.g. readout noise)

Dark Current in Infrared Materials

- Dark current is a function of temperature and cutoff wavelength.



Noise Sources: Thermal Noise

Properties of Johnson Noise

- due to random thermal motion of electric charge in conductors
- independent of current flow
- noise is random at all frequencies (up to ~170 fs), i.e. it is "white"

$$V_{n,thermal} = \sqrt{4kTBR}\ \{Volts\},$$

where k = Boltzmann's constant, $1.38(10^{-23})$ J/K,

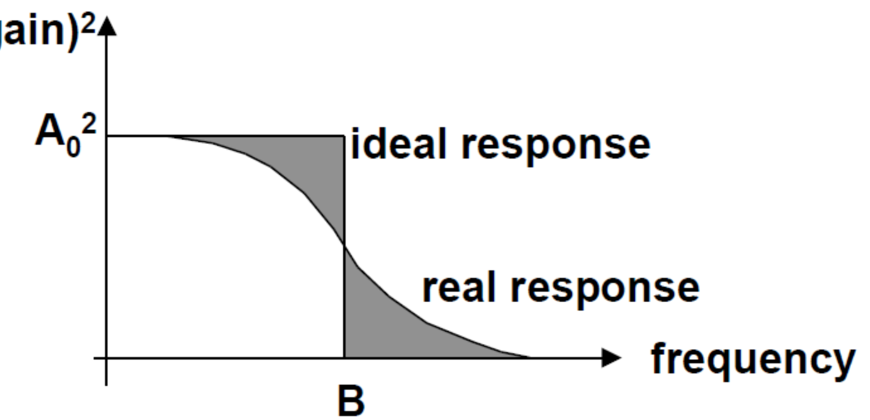
T = temperature (K),

B = bandwidth of system (Hz), and

R = resistance (Ohms).

Bandwidth

- The equivalent bandwidth of a circuit is described by the illustration below.



- The NEB is defined as the point at which the two shaded areas equal

- In the case of an RC circuit, the bandwidth is given by:

$$B = bandwidth = \frac{\pi f_{3dB}}{2} = \frac{\pi}{2(2\pi RC)} = \frac{1}{4RC}.$$

Noise Sources: kTC Noise

kTC Noise

- due to random thermal motion of electric charge in conductors, just like Johnson noise in resistors
- act of “resetting” capacitor freezes in random fluctuation of charge
- can be removed noiselessly through subtraction

$$V_{n,kTC} = \sqrt{kT / C}\ \{Volts\},$$

where k = Boltzmann’s constant, $1.38(10^{-23})$ J/K,

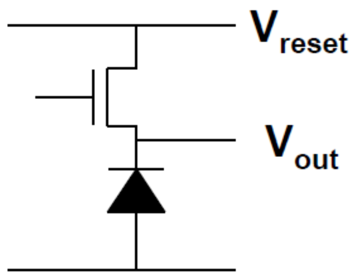
T = temperature (K), and

C = capacitance.

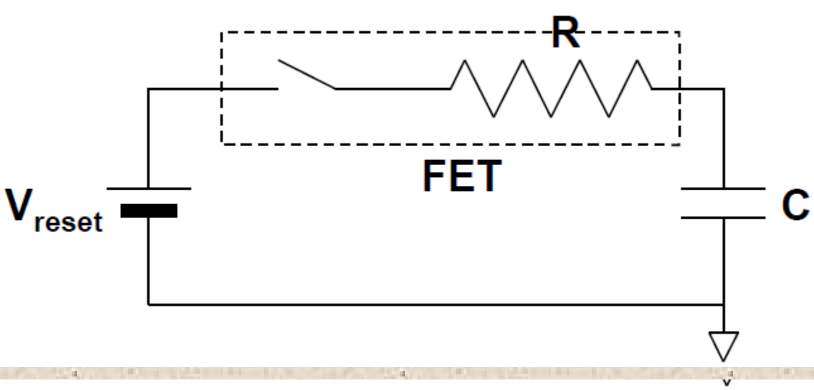
$$Q_{n,kTC} = \frac{CV_{n,kTC}}{e} = \frac{C\sqrt{kT / C}}{e} = \frac{\sqrt{kTC}}{e}\ \{\#e-\}.$$

kTC Noise as Reset Noise

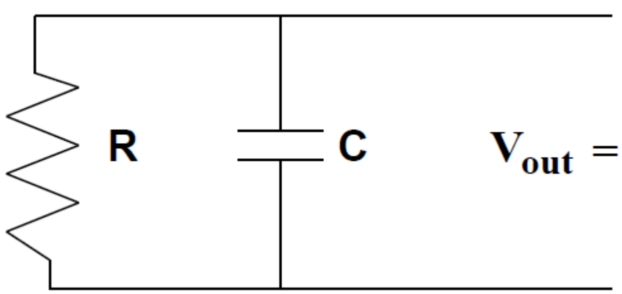
If we consider a diffusion (either a floating diffusion or a photodiode) being reset through a MOSFET



Effectively, this is a capacitance being charged through the resistance of the MOSFET channel



So the ac-equivalent circuit is



$$V_{out} = \sqrt{4kTBR}$$

From before, the bandwidth is

$$B = \frac{\pi}{2} f_0 = \frac{1}{4RC}$$

So we find the rms noise voltage

$$\langle v_{out} \rangle = \sqrt{\frac{kT}{C}}$$

Removing Reset Noise with CDS

- The reset noise adds an upredictable offset voltage to the signal.
- This offset can be removed by using correlated double sampling.

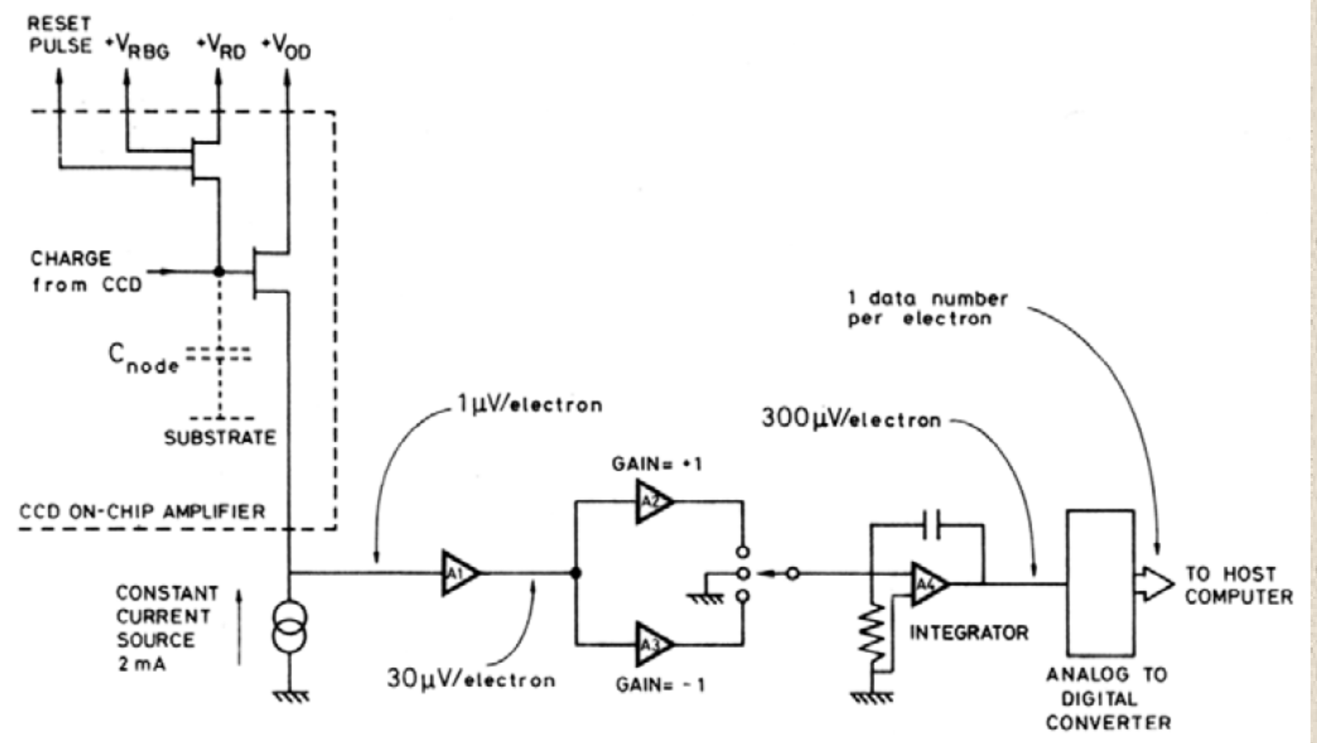


Figure 8.14. A block diagram of the principle of the correlated double-sampling (CDS) method of removing reset noise before the signal is digitized and sent to a computer. Credit: Craig Mackay.

Noise Sources: 1/f Noise

1/f noise

- sometimes called “flicker noise”
- caused by traps, often near surface interfaces
- occurs in most devices
- especially pronounced in FETs with small channels
- spectral density increases for lower frequencies

$$\overline{V_{1/f}^2} = \frac{K_f}{wLC_{ox}} \frac{\Delta f}{f} \{V^2\}, \textit{ where}$$

K_f is strongly dependent on technology, and

is typically ~ $3(10^{-24})$ V^2 F,

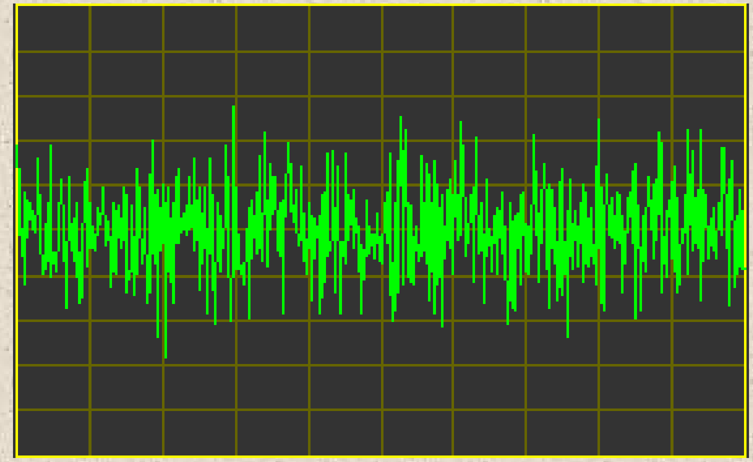
Δf = bandwidth of FET,

C_{ox} = capacitance of FET,

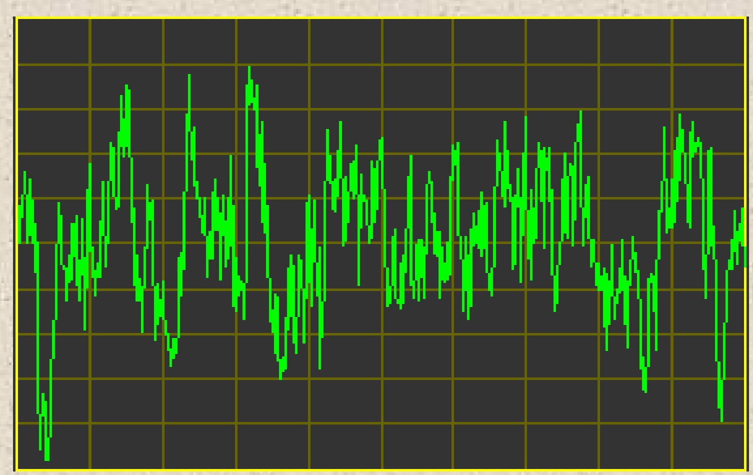
w = width of FET, and

L = length of FET.

Noise in Time Domain



white noise

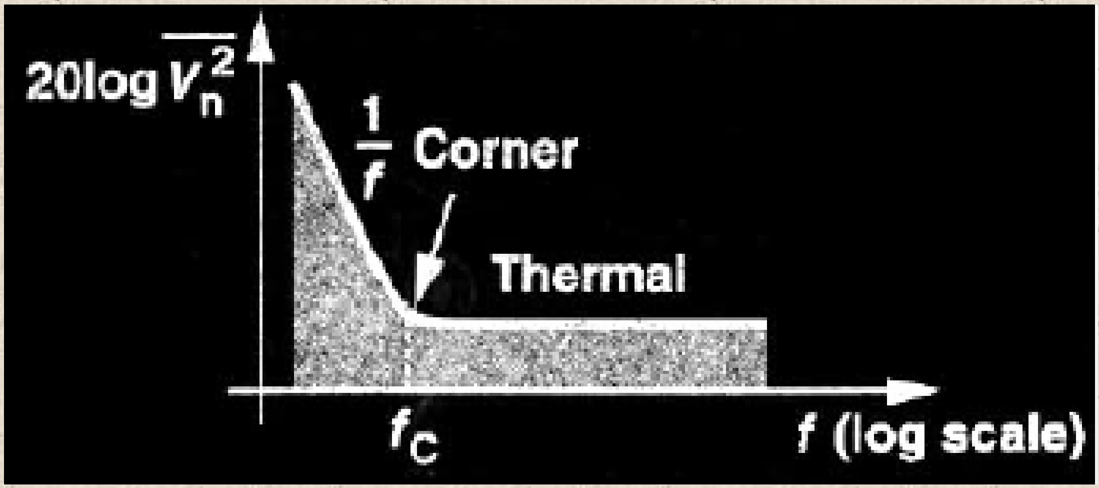


1/f noise

Noise in a MOSFET

- Noise is a combination of thermal and 1/f noise, with the latter dominating at low frequencies.

$$\frac{V_n^{\ 2}}{\Delta f} = \frac{V_{n,Johnson}^{\ \ 2}}{\Delta f} + \frac{V_{n,1/f}^{\ \ 2}}{\Delta f} = \frac{4kT}{g_m} + \frac{K}{wlC_{ox}}\frac{1}{f}.$$



Noise Sources: Popcorn Noise

Popcorn Noise

- ~~This is the minimum noise you can hear in a movie theatre during a tense scene.~~
- This is noise produced by traps in FET channels that temporarily change the properties of the channel.
- The summation of traps is thought to be a potential source of 1/f noise.

Popcorn noise, sometimes called burst noise or random-telegraph-signal (RTS) noise, is a discrete modulation of the channel current caused by the capture and emission of a channel carrier. See Figure 3.

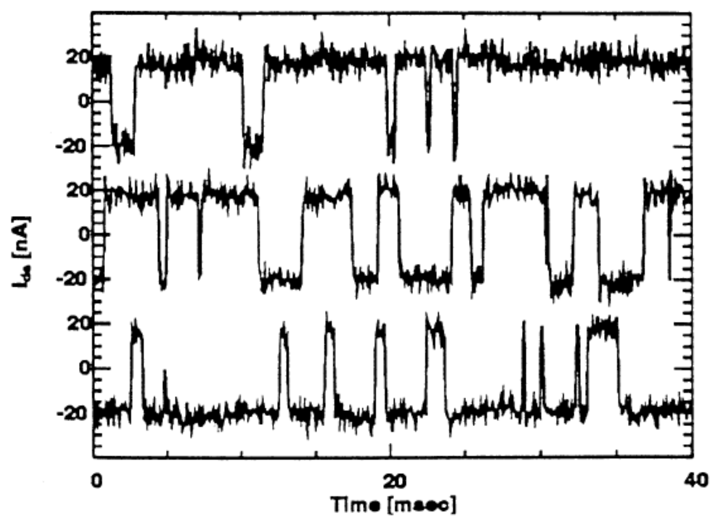


Figure 3: Typical popcorn noise, showing discrete levels of channel current modulation due to the trapping and release of a single carrier, for three different bias conditions, from [30, Figure 1].

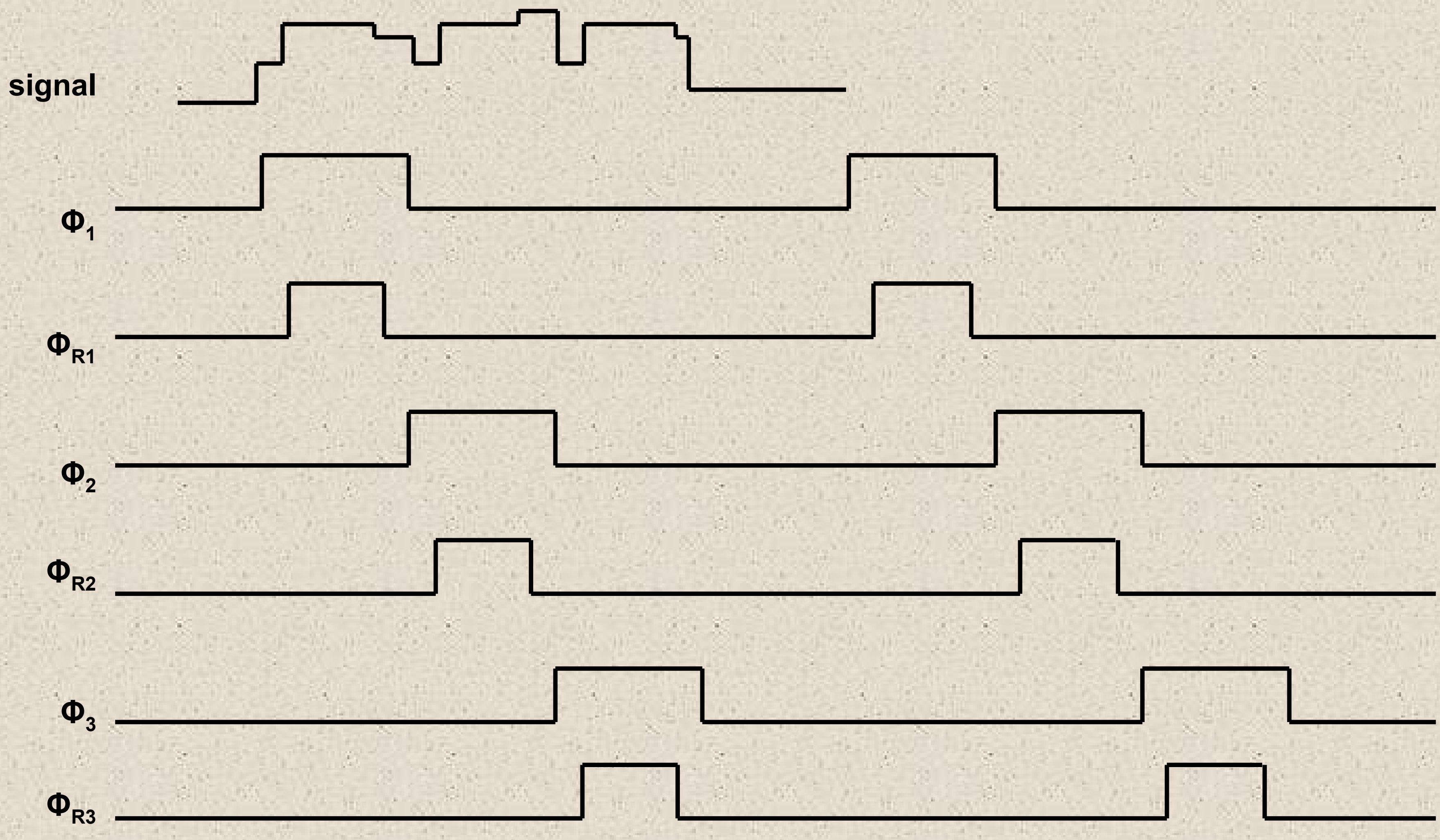
Read Noise

Read Noise

- Read noise is produced by all the electronics in a detector system, e.g.
  - Johnson noise,
  - 1/f noise,
  - electronic current shot noise,
  - unstable power supplies
- It is usually measured as the standard deviation in a sample of multiple reads taken with minimum exposure time and under dark conditions.

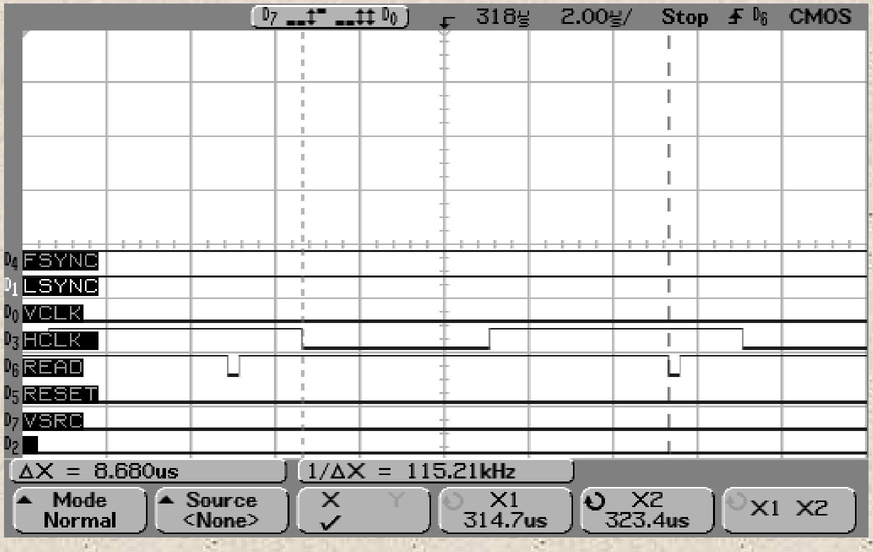
Noise Sources: Electronic Crosstalk

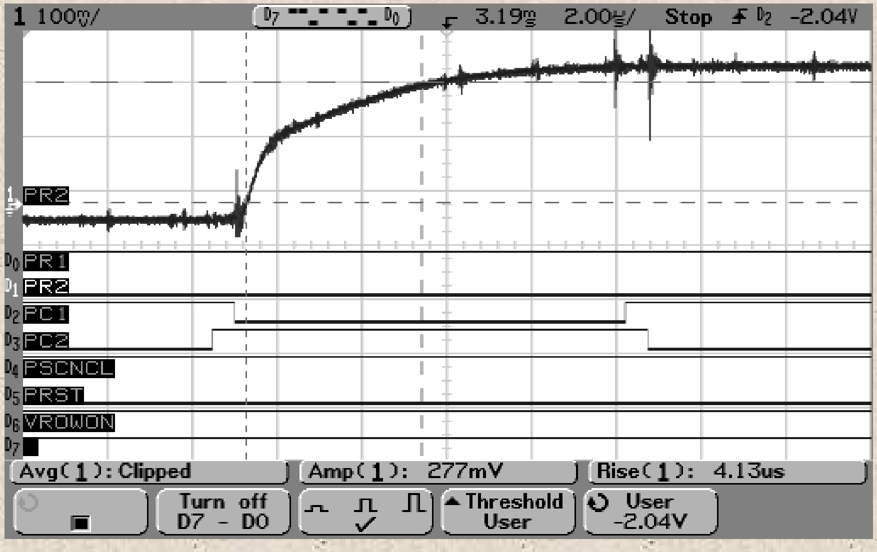
Electronic Crosstalk



Clocking Feedthrough (Crosstalk)

- read time, 10.18 us
- wait time for convert, 8.7 us
- bandwidth ~ 160 kHz

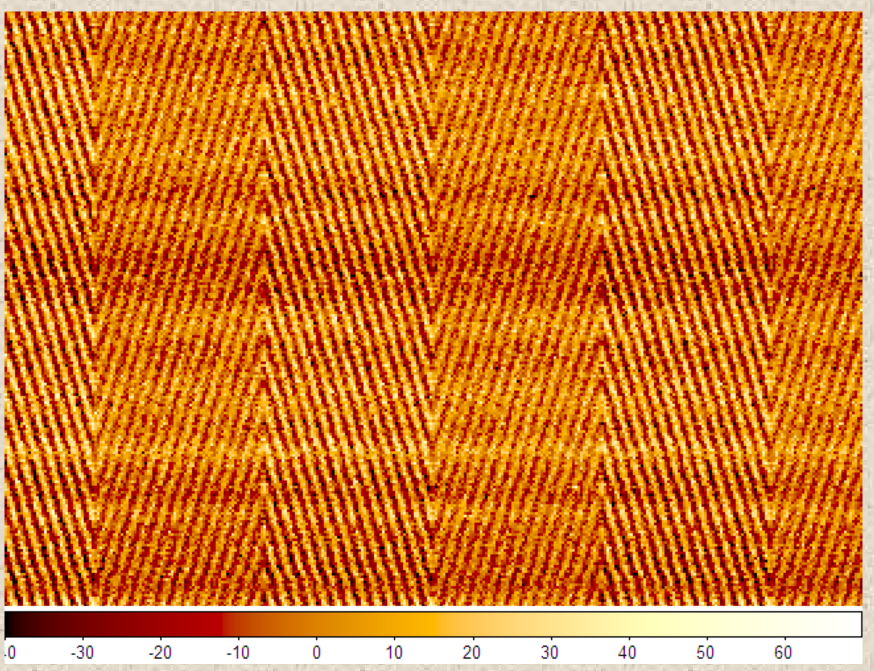




Noise Sources: Pickup or Interference

Pickup or Interference

- This noise is produced by ambient electromagnetic fields from nearby radiators.
- Interference doesn't have to be periodic, but it often is.
- As an example, consider the image below which shows ~20kHz interference pattern in a small section of a full 2Kx2K detector readout.



Improving SNR

SNR

- SNR can be improved by maximizing the numerator and/or minimizing the denominator of the full SNR equation.
- The choice of what to optimize often comes down to money. That is, some things are expensive to improve and some are not.
- For instance, the background flux can be reduced in astrophysics applications by launching the system into space (for the cost of billions of dollars….).

$$SNR = \frac{S}{N} = \frac{\eta_{inst} A \frac{\Delta\nu}{h\nu} F_\nu t QE_\nu}{\sqrt{\left(\eta_{inst} A \frac{\Delta\nu}{h\nu} F_\nu t QE_\nu\right) + \left(\eta_{inst} A \frac{\Delta\nu}{h\nu} F_{back,\nu} t QE_\nu\right) + i_{dark} t + N_{read}^2}}.$$

Improving SNR

- Optical effects
  - Throughput: bigger aperture, anti-reflection coatings
  - Background: low scatter materials, cooling
- Detector effects
  - Dark current: high purity material, low surface leakage
  - Read Noise: multiple sampling, in-pixel digitization, photon-counting
  - QE: thickness optimization, anti-reflection coatings, depleted
- Atmospheric effects
  - Atmospheric absorption: higher altitude
  - OH emission: OH suppression instruments
  - Turbulence: adaptive optics
  - Ultimate "fix" is to go to space!

Increasing Integration Time/Coadds

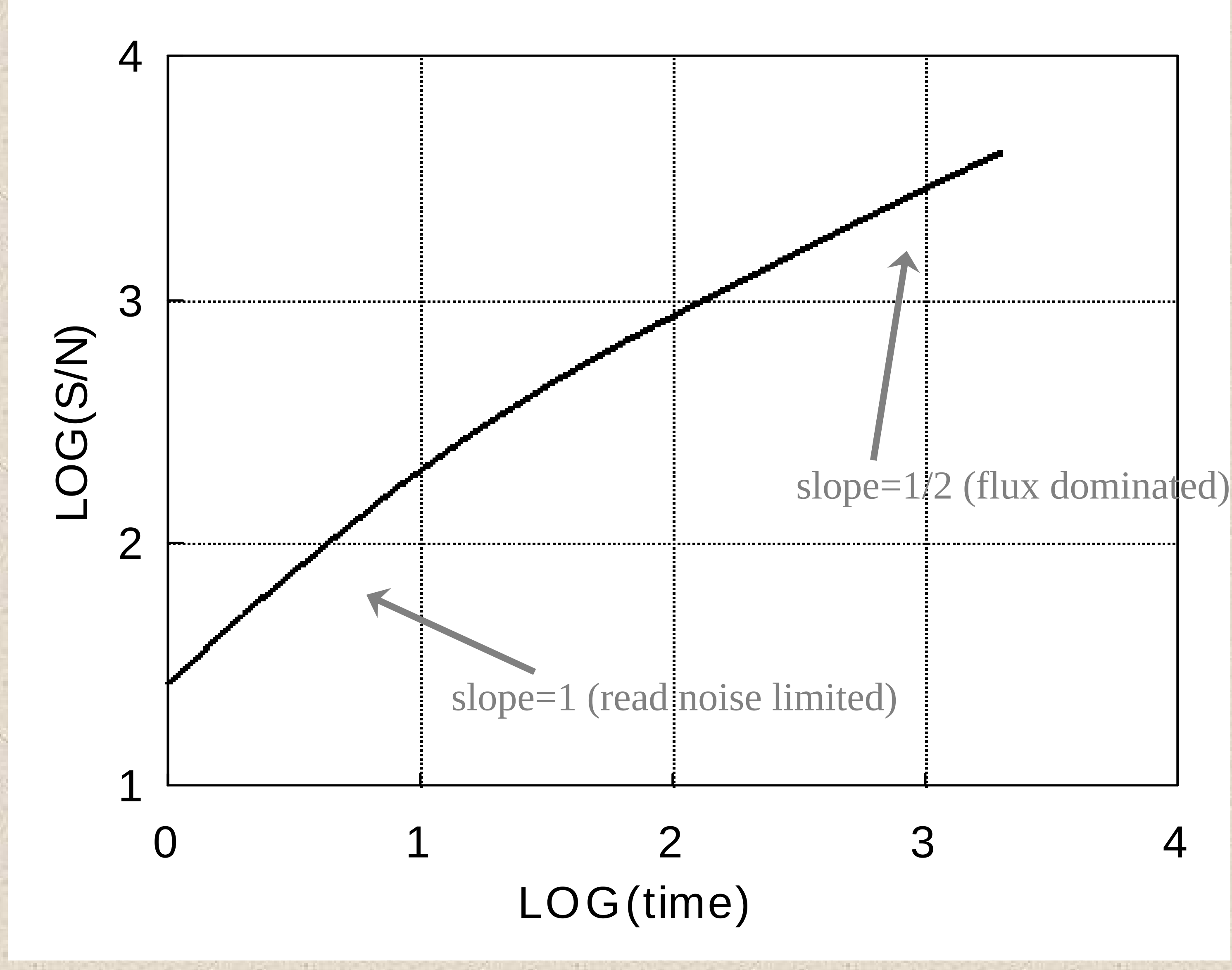
- Signal increases with exposure time. Noise can also increase, but not by as much.
- "Coadding" is summing individual exposures – similar to increasing exposure time.

$$SNR = \frac{S}{N} = \frac{\eta_{inst} A \frac{\Delta\nu}{h\nu} F_\nu t QE_\nu}{\sqrt{\left(\eta_{inst} A \frac{\Delta\nu}{h\nu} F_\nu t QE_\nu\right) + \left(\eta_{inst} A \frac{\Delta\nu}{h\nu} F_{back,\nu} t QE_\nu\right) + i_{dark} t + N_{read}^2}}.$$

$$SNR \xrightarrow{shot\ noise} \sqrt{\eta_{inst} A \frac{\Delta\nu}{h\nu} F_\nu t QE_\nu} \propto \sqrt{t}.$$

$$SNR \xrightarrow{read\ noise} \frac{\eta_{inst} A \frac{\Delta\nu}{h\nu} F_\nu t QE_\nu}{N_{read}} \propto t.$$

Read noise vs. Shot Noise Limited Case



Fowler Sampling

- Fowler sampling uses the averages of groups of non-destructive reads at the beginning and end of exposure.
- This sampling mode generally reduces random noise by the square root of the number of reads.

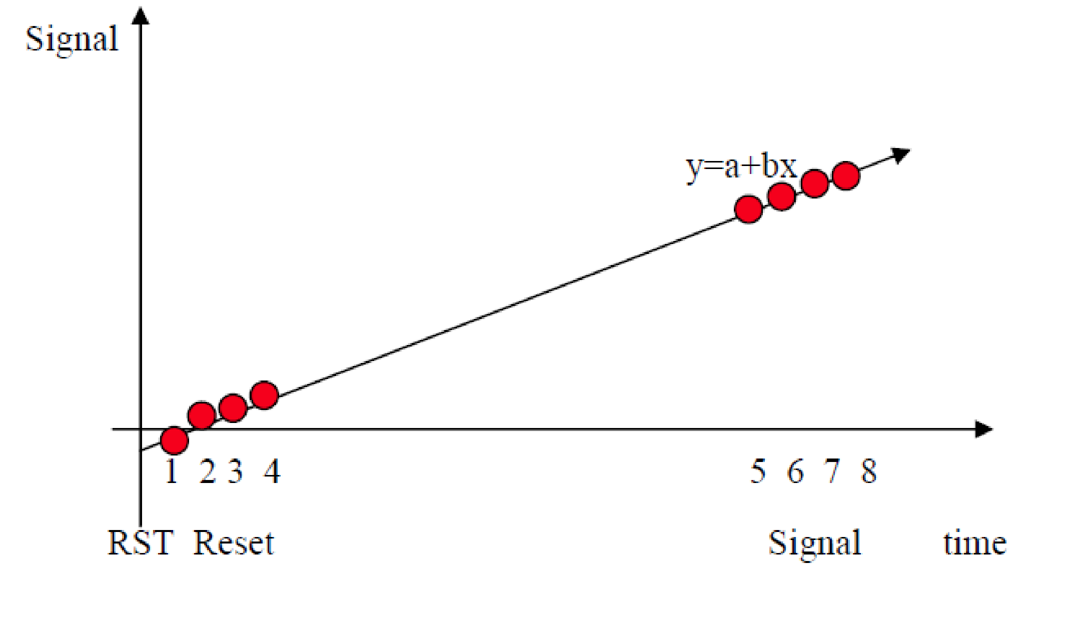
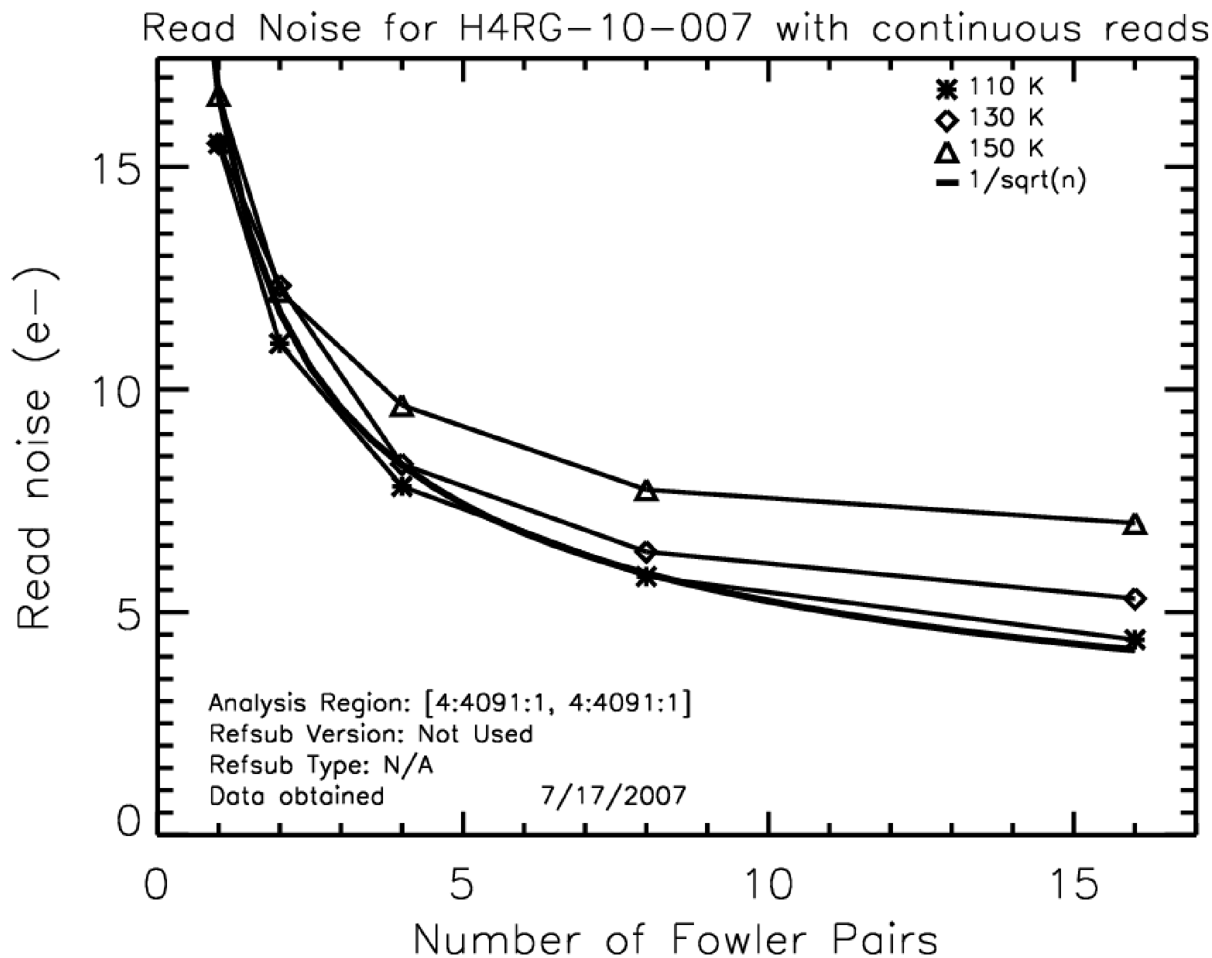


Figure 2. Sketch of the Fowler sampling readout scheme.

http://www.stsci.edu/hst/wfc3/documents/ISRs/WFC3-2007-12.pdf

Improving SNR: multiple sampling



Up-the-ramp Sampling

- In up-the-ramp sampling, the signal is non-destructively read out many times during an exposure.
- This read mode generally reduces random noise by the square root of the number of reads.
- The math can be more difficult than for Fowler sampling.
- This mode is potentially good for removing cosmic rays.

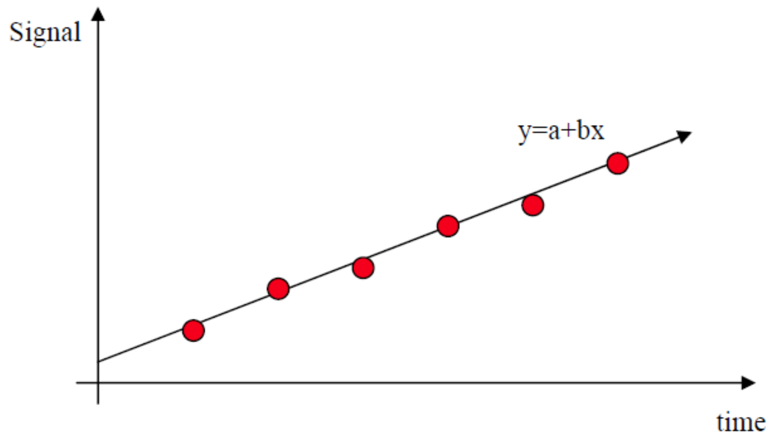


Figure 1. Sketch of the linear accumulation of the signal and readout in the sampling-up-the-ramp mode.

Read Noise Limited Case

- In the read noise limited case, SNR can be improved through multiple non-desctructive reads.
- However, some amount of exposure time is lost by performing many reads, so there is a tradeoff that depends on how long it takes to do a read.

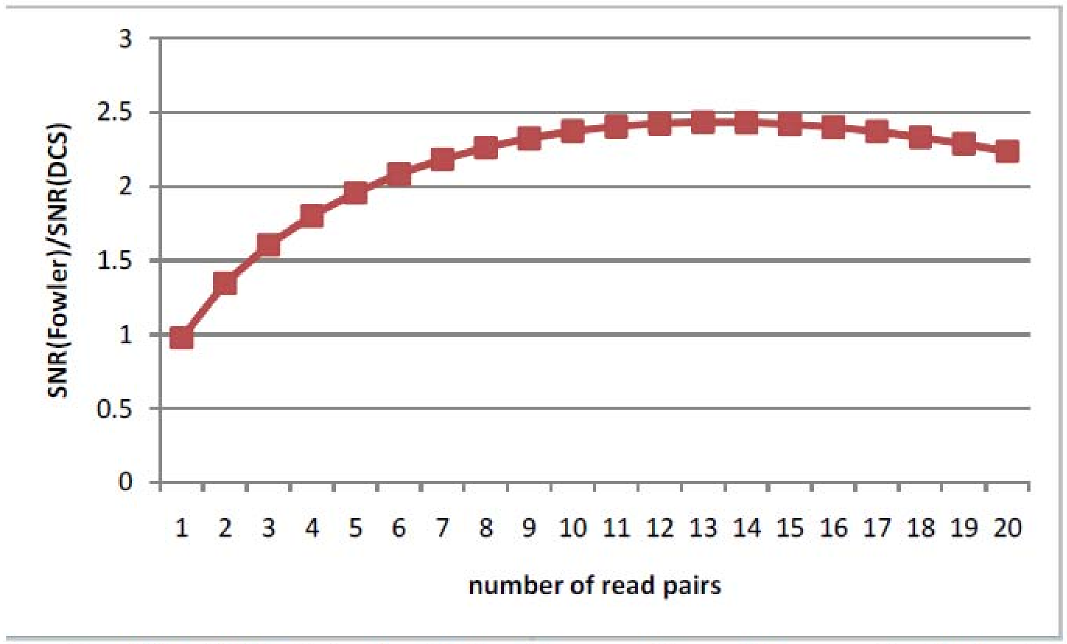


Figure 3. ratio of the SNR attained with Fowler sampling with respect to the basic Double Correlated Sampling.

Background Limited Case

- In the background limited case, it does not help to obtain multiple samples. It just wastes time and reduces exposure time.

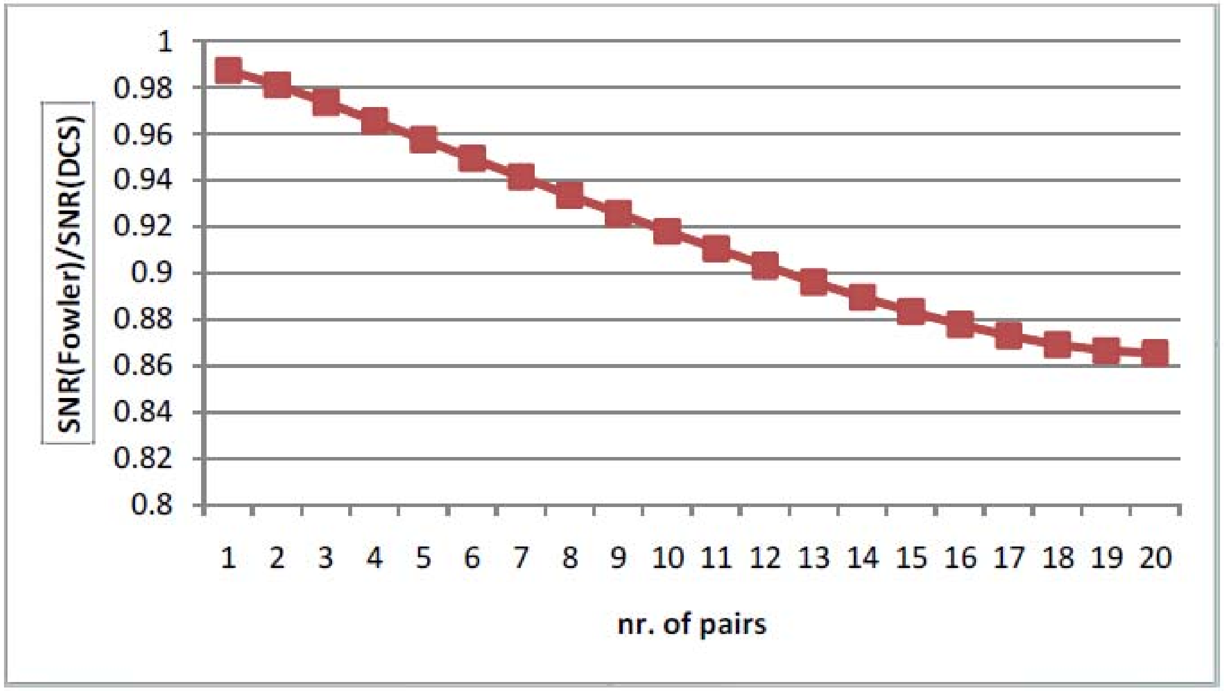


Figure 4. Ration of the SNR attained with the Fowler sampling with respect to the basic Double Correlated Sampling in Poisson limited regime.

Sampling Mode Summary: CDS

DOUBLE CORRELATE SAMPLING

SIGNAL:

$$S = F \cdot T_{\text{int}}$$

NOISE

RON LIMITED

$$\sigma_S^2 = 2 \cdot \sigma_{ron}^2$$

POISSON LIMITED

$$\sigma_S^2 = F \cdot T_{\text{int}}$$

SIGNAL TO NOISE RATIO

$$SNR_{dcs,ron} = \frac{F \cdot T_{\text{int}}}{\sqrt{2} \cdot \sigma_{ron}}$$

$$SNR_{dcs,Pois.} = \sqrt{F \cdot T_{\text{int}}}$$

TOTAL SIGNAL TO NOISE RATIO

$$SNR_{dcs} = \frac{F \cdot T_{\text{int}}}{\left[2 \cdot \sigma_{ron}^2 + F \cdot T_{\text{int}}\right]^{1/2}}$$

Sampling Mode Summary: Up-the-ramp

LINE-FITTING

SIGNAL:

$$S_N = F \cdot T_{\text{int}}(N)$$

NOISE

RON LIMITED

$$\sigma^2_{S_N} = \frac{12 \cdot N}{N^2 - 1}\sigma^2_{ron}$$

POISSON LIMITED

$$\sigma^2_{S_N} = \frac{6}{5}\frac{N^2+1}{N^2-1}F \cdot T_{\text{int}}(N)$$

SIGNAL TO NOISE RATIO

$$SNR_{LF,ron} = \sqrt{\frac{N(N+1)}{6(N-1)}}SNR_{DCS,ron}$$

$$SNR_{LF,Pois.} = \sqrt{\frac{5}{6}\left(\frac{N^2-1}{N^2+1}\right)}SNR_{dcs,Pois.}$$

TOTAL SIGNAL TO NOISE RATIO

$$SNR_{LF} = \frac{F \cdot T_{\text{int}}(N)}{\left[\frac{12(N-1)}{N(N+1)} \cdot \sigma^2_{ron} + \frac{6}{5}\frac{N^2+1}{N^2-1}F \cdot T_{\text{int}}(N)\right]^{1/2}}$$

Sampling Mode Summary: Fowler

FOWLER SAMPLING

SIGNAL:

$$S = F \cdot T_{eff} = F \cdot \left(T_{\text{int}} - N_p \cdot dt\right)$$

NOISE

RON LIMITED

$$\sigma_S^2 = \frac{2 \cdot \sigma_{ron}^2}{N_p}$$

POISSON LIMITED

$$\sigma_S^2 = F \cdot T_{\text{int}} + F \cdot dt \left(\frac{1}{3N_p} - \frac{4N_p}{3} \right)$$

SIGNAL TO NOISE RATIO

$$SNR_{Fowler,RO} = SNR_{DCS} \sqrt{N_p} \left(1 - N_p \frac{dt}{T_{\text{int}}} \right)$$

$$SNR_{Fowler,Pois.} = SNR_{dcs,Pois.} \frac{1 - N_p \dfrac{dt}{T_{\text{int}}}}{\sqrt{1 + \dfrac{dt}{3T_{\text{int}}} \left(\dfrac{1}{N_p} - 4N_p \right)}}$$

TOTAL SIGNAL TO NOISE RATIO

$$SNR_{Fowler} = \frac{F \cdot \left(T_{\text{int}} - n_p dt\right)}{\left[\dfrac{2 \cdot \sigma_{ron}^2}{n_p} + F \cdot T_{\text{int}} + F \cdot dt \left(\dfrac{1}{3n_p} - \dfrac{4n_p}{3} \right) \right]^{1/2}}$$

Other Electronic Techniques

- bandwidth limiting filters
- increasing gain (before noise is injected)
- reducing the unit cell capacitance (thereby increasing in-pixel gain).